Spring 2012

EE 330

ENGINEERING ELECTROMAGNETICS

HW 5: *Due Friday 10 February*
2.29, 2.74, 3.26, 3.28, 3.38, 3.45, 3.55, 3.57,
4.5c, 4.9, 4.11, 4.17, 4.22, 4.27, 4.36

Problem 2.29 Show that the input impedance of a quarter-wavelength–long lossless line terminated in a short circuit appears as an open circuit.

Solution:

$$Z_{\text{in}} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right).$$

For $l = \frac{\lambda}{4}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$. With $Z_L = 0$, we have

$$Z_{\text{in}} = Z_0 \left(\frac{jZ_0 \tan \pi/2}{Z_0} \right) = j\infty \quad \text{(open circuit).}$$

Problem 2.74 A 25-Ω antenna is connected to a 75-Ω lossless transmission line. Reflections back toward the generator can be eliminated by placing a shunt impedance Z at a distance l from the load (Fig. P2.74). Determine the values of Z and l.

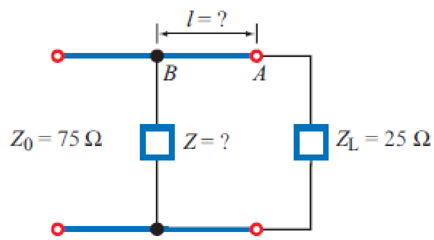
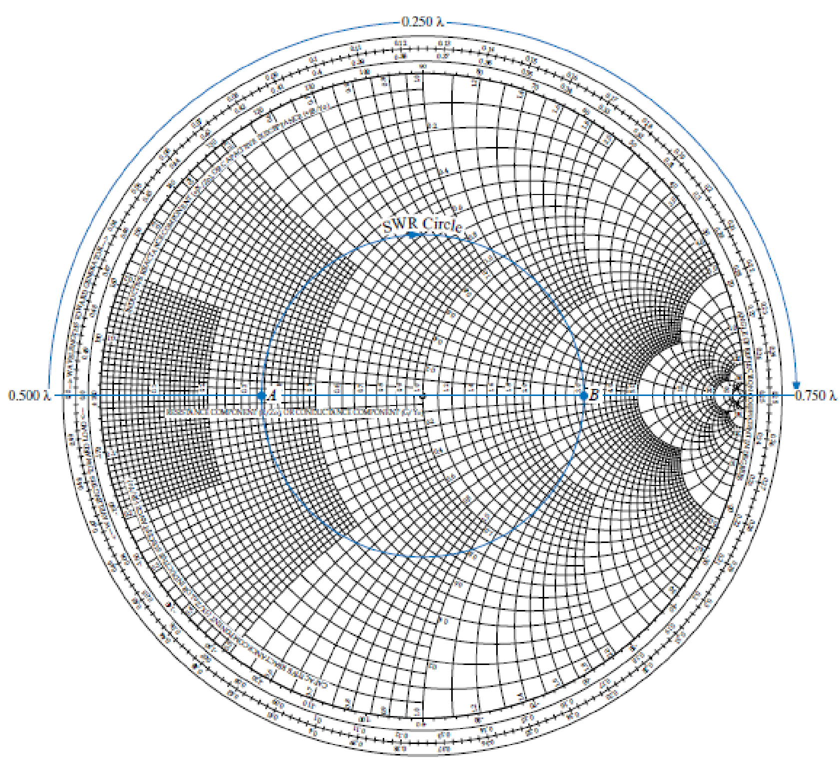


Figure P2.74: Circuit for Problem 2.74.

Solution:



The normalized load impedance is:

$$z_L = \frac{25}{75} = 0.33 \qquad \text{(point } A \text{ on Smith chart)}$$

The Smith chart shows A and the SWR circle. The goal is to have an equivalent impedance of 75 Ω to the left of B. That equivalent impedance is the parallel combination of Z_{in} at B (to the right of the shunt impedance Z) and the shunt element Z. Since we need for this to be purely real, it's best to choose l such that Z_{in} is purely real, thereby choosing Z to be simply a resistor. Adding two resistors in parallel generates a sum smaller in magnitude than either one of them. So we need for Z_{in} to be larger than Z_0, not smaller. On the Smith chart, that point is B, at a distance $l = \lambda/4$ from the load. At that point:

$$z_{in} = 3,$$

which corresponds to

$$y_{in} = 0.33.$$

Hence, we need y, the normalized admittance corresponding to the shunt impedance Z, to have a value that satisfies:

$$\begin{aligned} y_{in} + y &= 1 \\ y &= 1 - y_{in} = 1 - 0.33 = 0.66 \\ z &= \frac{1}{y} = \frac{1}{0.66} = 1.5 \\ Z &= 75 \times 1.5 = 112.5\ \Omega. \end{aligned}$$

In summary,

$$\begin{aligned} l &= \frac{\lambda}{4}, \\ Z &= 112.5\ \Omega. \end{aligned}$$

Problem 3.26 Find the volumes described by
(a) $2 \leq r \leq 5;\ \pi/2 \leq \phi \leq \pi;\ 0 \leq z \leq 2,$
(b) $0 \leq R \leq 5;\ 0 \leq \theta \leq \pi/3;\ 0 \leq \phi \leq 2\pi.$
Also sketch the outline of each volume.

Solution:

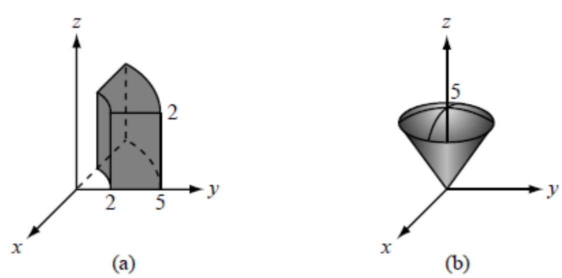


Figure P3.26: Volumes described by Problem 3.26 .

(a) From Eq. (3.44),

$$\mathcal{V} = \int_{z=0}^{2} \int_{\phi=\pi/2}^{\pi} \int_{r=2}^{5} r\,dr\,d\phi\,dz = \left(\left(\left(\tfrac{1}{2} r^2 \phi z \right) \Big|_{r=2}^{5} \right) \Big|_{\phi=\pi/2}^{\pi} \right) \Bigg|_{z=0}^{2} = \frac{21\pi}{2}.$$

(b) From Eq. (3.50e),

$$\begin{aligned} \mathcal{V} &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/3} \int_{R=0}^{5} R^2 \sin\theta\, dR\, d\theta\, d\phi \\ &= \left(\left(\left(-\cos\theta \frac{R^3}{3} \phi \right) \Big|_{R=0}^{5} \right) \Bigg|_{\theta=0}^{\pi/3} \right) \Bigg|_{\phi=0}^{2\pi} = \frac{125\pi}{3}. \end{aligned}$$

Problem 3.28 A vector field is given in cylindrical coordinates by

$$\mathbf{E} = \hat{\mathbf{r}}r\cos\phi + \hat{\boldsymbol{\phi}}r\sin\phi + \hat{\mathbf{z}}z^2.$$

Point $P = (2, \pi, 3)$ is located on the surface of the cylinder described by $r = 2$. At point P, find:

(a) the vector component of $\mathbf{E}$ perpendicular to the cylinder,

(b) the vector component of $\mathbf{E}$ tangential to the cylinder.

Solution:

(a) $\mathbf{E}_n = \hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{E}) = \hat{\mathbf{r}}[\hat{\mathbf{r}}\cdot(\hat{\mathbf{r}}r\cos\phi + \hat{\boldsymbol{\phi}}r\sin\phi + \hat{\mathbf{z}}z^2)] = \hat{\mathbf{r}}r\cos\phi$.

At $P = (2, \pi, 3)$, $\mathbf{E}_n = \hat{\mathbf{r}}2\cos\pi = -\hat{\mathbf{r}}2$.

(b) $\mathbf{E}_t = \mathbf{E} - \mathbf{E}_n = \hat{\boldsymbol{\phi}}r\sin\phi + \hat{\mathbf{z}}z^2$.

At $P = (2, \pi, 3)$, $\mathbf{E}_t = \hat{\boldsymbol{\phi}}2\sin\pi + \hat{\mathbf{z}}3^2 = \hat{\mathbf{z}}9$.

Problem 3.38 The gradient of a scalar function T is given by

$$\nabla T = \hat{\mathbf{z}}e^{-3z}.$$

If $T = 10$ at $z = 0$, find $T(z)$.

Solution:

$$\nabla T = \hat{\mathbf{z}}e^{-3z}.$$

By choosing P_1 at $z = 0$ and P_2 at any point z, (3.76) becomes

$$\begin{aligned} T(z) - T(0) &= \int_0^z \nabla T \cdot d\mathbf{l}' = \int_0^z \hat{\mathbf{z}}e^{-3z'} \cdot (\hat{\mathbf{x}}\,dx' + \hat{\mathbf{y}}\,dy' + \hat{\mathbf{z}}\,dz') \\ &= \int_0^z e^{-3z'}\,dz' = -\frac{-e^{-3z'}}{3}\bigg|_0^z = \frac{1}{3}(1 - e^{-3z}). \end{aligned}$$

Hence,

$$T(z) = T(0) + \frac{1}{3}(1 - e^{-3z}) = 10 + \frac{1}{3}(1 - e^{-3z}).$$

Problem 3.45 Vector field $\mathbf{E}$ is characterized by the following properties: (a) $\mathbf{E}$ points along $\hat{\mathbf{R}}$, (b) the magnitude of $\mathbf{E}$ is a function of only the distance from the origin, (c) $\mathbf{E}$ vanishes at the origin, and (d) $\nabla\cdot\mathbf{E} = 12$, everywhere. Find an expression for $\mathbf{E}$ that satisfies these properties.

Solution: According to properties (a) and (b), $\mathbf{E}$ must have the form

$$\mathbf{E} = \hat{\mathbf{R}}E_R$$

where E_R is a function of R only.

$$\begin{aligned} \nabla\cdot\mathbf{E} &= \frac{1}{R^2}\frac{\partial}{\partial R}(R^2E_R) = 12, \\ \frac{\partial}{\partial R}(R^2E_R) &= 12R^2, \\ \int_0^R \frac{\partial}{\partial R}(R^2E_R)\,dR &= \int_0^R 12R^2\,dR, \\ R^2E_R\big|_0^R &= \frac{12R^3}{3}\bigg|_0^R, \\ R^2E_R &= 4R^3. \end{aligned}$$

Hence,

$$E_R = 4R,$$

and

$$\mathbf{E} = \hat{\mathbf{R}}4R.$$

Problem 3.55 Verify Stokes's theorem for the vector field $\mathbf{B} = (\hat{\mathbf{r}}\cos\phi + \hat{\boldsymbol{\phi}}\sin\phi)$ by evaluating:

(a) $\oint_C \mathbf{B}\cdot d\boldsymbol{\ell}$ over the path comprising a quarter section of a circle, as shown in Fig. P3.55, and

(b) $\int_S (\nabla\times\mathbf{B})\cdot d\mathbf{s}$ over the surface of the quarter section.

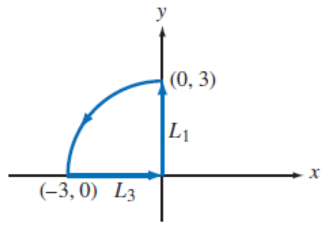


Figure P3.55: Problem 3.55.

Solution:

(a)

$$\oint_C \mathbf{B}\cdot d\boldsymbol{\ell} = \int_{L_1} \mathbf{B}\cdot d\boldsymbol{\ell} + \int_{L_2} \mathbf{B}\cdot d\boldsymbol{\ell} + \int_{L_3} \mathbf{B}\cdot d\boldsymbol{\ell}$$

Given the shape of the path, it is best to use cylindrical coordinates. $\mathbf{B}$ is already expressed in cylindrical coordinates, and we need to choose $d\boldsymbol{\ell}$ in cylindrical coordinates:

$$d\boldsymbol{\ell} = \hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz.$$

Along path L_1, $d\phi = 0$ and $dz = 0$. Hence, $d\boldsymbol{\ell} = \hat{\mathbf{r}}\,dr$ and

$$\begin{aligned}\int_{L_1} \mathbf{B}\cdot d\boldsymbol{\ell} &= \int_{r=0}^{r=3} (\hat{\mathbf{r}}\cos\phi + \hat{\boldsymbol{\phi}}\sin\phi)\cdot\hat{\mathbf{r}}\,dr\Big|_{\phi=90^\circ}\\ &= \int_{r=0}^{3} \cos\phi\,dr\Big|_{\phi=90^\circ} = r\cos\phi|_{r=0}^{3}\Big|_{\phi=90^\circ} = 0.\end{aligned}$$

Along L_2, $dr = dz = 0$. Hence, $d\boldsymbol{\ell} = \hat{\boldsymbol{\phi}}r\,d\phi$ and

$$\begin{aligned}\int_{L_2} \mathbf{B}\cdot d\boldsymbol{\ell} &= \int_{\phi=90^\circ}^{180^\circ} (\hat{\mathbf{r}}\cos\phi + \hat{\boldsymbol{\phi}}\sin\phi)\cdot\hat{\boldsymbol{\phi}}r\,d\phi\Big|_{r=3}\\ &= -3\cos\phi|_{90^\circ}^{180^\circ} = 3.\end{aligned}$$

Along L_3, $dz = 0$ and $d\phi = 0$. Hence, $d\boldsymbol{\ell} = \hat{\mathbf{r}}\,dr$ and

$$\begin{aligned}\int_{L_3} \mathbf{B}\cdot d\boldsymbol{\ell} &= \int_{r=3}^{0} (\hat{\mathbf{r}}\cos\phi + \hat{\boldsymbol{\phi}}\sin\phi)\cdot\hat{\mathbf{r}}\,dr\Big|_{\phi=180^\circ}\\ &= \int_{r=3}^{0} \cos\phi\,dr|_{\phi=180^\circ} = -r|_{3}^{0} = 3.\end{aligned}$$

Hence,

$$\oint_C \mathbf{B}\cdot d\boldsymbol{\ell} = 0 + 3 + 3 = 6.$$

(b)

$$\begin{aligned}\nabla\times\mathbf{B} &= \hat{\mathbf{z}}\frac{1}{r}\left(\frac{\partial}{\partial r}\left(rB_\phi - \frac{\partial B_r}{\partial\phi}\right)\right)\\ &= \hat{\mathbf{z}}\frac{1}{r}\left(\frac{\partial}{\partial r}(r\sin\phi) - \frac{\partial}{\partial\phi}(\cos\phi)\right)\\ &= \hat{\mathbf{z}}\frac{1}{r}(\sin\phi + \sin\phi) = \hat{\mathbf{z}}\frac{2}{r}\sin\phi.\end{aligned}$$

$$\begin{aligned}\int_S (\nabla\times\mathbf{B})\cdot d\mathbf{s} &= \int_{r=0}^{3}\int_{\phi=90^\circ}^{180^\circ} \left(\hat{\mathbf{z}}\frac{2}{r}\sin\phi\right)\cdot\hat{\mathbf{z}}r\,dr\,d\phi\\ &= -2r|_{r=0}^{3}\cos\phi\Big|_{\phi=90^\circ}^{180^\circ} = 6.\end{aligned}$$

Hence, Stokes's theorem is verified.

Problem 3.57 Find the Laplacian of the following scalar functions:

(a) $V = 4xy^2z^3$,

(b) $V = xy + yz + zx$,

(c) $V = 3/(x^2 + y^2)$,

(d) $V = 5e^{-r}\cos\phi$,

(e) $V = 10e^{-R}\sin\theta$.

Solution:

(a) From Eq. (3.110), $\nabla^2(4xy^2z^3) = 8xz^3 + 24xy^2z$.

(b) $\nabla^2(xy + yz + zx) = 0$.

(c) From the inside back cover of the book,

$$\nabla^2\left(\frac{3}{x^2+y^2}\right) = \nabla^2(3r^{-2}) = 12r^{-4} = \frac{12}{(x^2+y^2)^2}.$$

(d)

$$\nabla^2(5e^{-r}\cos\phi) = 5e^{-r}\cos\phi\left(1 - \frac{1}{r} - \frac{1}{r^2}\right).$$

(e)

$$\nabla^2(10e^{-R}\sin\theta) = 10e^{-R}\left[\sin\theta\left(1 - \frac{2}{R}\right) + \frac{\cos^2\theta - \sin^2\theta}{R^2\sin\theta}\right].$$

Problem 4.5 Find the total charge on a circular disk defined by $r \le a$ and $z = 0$ if:

(a) $\rho_s = \rho_{s0}\cos\phi$ (C/m^2)

(b) $\rho_s = \rho_{s0}\sin^2\phi$ (C/m^2)

(c) $\rho_s = \rho_{s0}e^{-r}$ (C/m^2)

(d) $\rho_s = \rho_{s0}e^{-r}\sin^2\phi$ (C/m^2)

where ρ_{s0} is a constant.

Solution:

(a)

$$Q = \int \rho_s\,ds = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0}\cos\phi\;r\,dr\,d\phi = \rho_{s0}\left.\frac{r^2}{2}\right|_0^a \left.\sin\phi\right|_0^{2\pi} = 0.$$

(b)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0}\sin^2\phi\;r\,dr\,d\phi = \rho_{s0}\left.\frac{r^2}{2}\right|_0^a \int_0^{2\pi}\left(\frac{1-\cos 2\phi}{2}\right)d\phi$$
$$= \frac{\rho_{s0}a^2}{4}\left(\phi - \frac{\sin 2\phi}{2}\right)\Bigg|_0^{2\pi} = \frac{\pi a^2}{2}\rho_{s0}.$$

(c)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0}e^{-r}r\,dr\,d\phi = 2\pi\rho_{s0}\int_0^a re^{-r}\,dr$$
$$= 2\pi\rho_{s0}\left[-re^{-r} - e^{-r}\right]_0^a$$
$$= 2\pi\rho_{s0}[1 - e^{-a}(1+a)].$$

(d)

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} \rho_{s0}e^{-r}\sin^2\phi\;r\,dr\,d\phi$$
$$= \rho_{s0}\int_{r=0}^{a} re^{-r}\,dr \int_{\phi=0}^{2\pi}\sin^2\phi\,d\phi$$
$$= \rho_{s0}[1 - e^{-a}(1+a)]\cdot\pi = \pi\rho_{s0}[1 - e^{-a}(1+a)].$$

Problem 4.9 A circular beam of charge of radius a consists of electrons moving with a constant speed u along the $+z$-direction. The beam's axis is coincident with the z-axis and the electron charge density is given by

$$\rho_v = -cr^2 \quad (\text{c/m}^3)$$

where c is a constant and r is the radial distance from the axis of the beam.

(a) Determine the charge density per unit length.

(b) Determine the current crossing the z-plane.

Solution:

(a)

$$\begin{aligned}
\rho_l &= \int \rho_v \, ds \\
&= \int_{r=0}^{a} \int_{\phi=0}^{2\pi} -cr^2 \cdot r \, dr \, d\phi = -2\pi c \left. \frac{r^4}{4} \right|_0^a = -\frac{\pi c a^4}{2} \quad (\text{C/m}).
\end{aligned}$$

(b)

$$\begin{aligned}
\mathbf{J} &= \rho_v \mathbf{u} = -\hat{\mathbf{z}} cr^2 u \quad (\text{A/m}^2) \\
I &= \int \mathbf{J} \cdot d\mathbf{s} \\
&= \int_{r=0}^{a} \int_{\phi=0}^{2\pi} (-\hat{\mathbf{z}} cur^2) \cdot \hat{\mathbf{z}} r \, dr \, d\phi \\
&= -2\pi cu \int_0^a r^3 \, dr = -\frac{\pi c u a^4}{2} = \rho_l u. \quad (\text{A}).
\end{aligned}$$

Problem 4.11 A square with sides of 2 m has a charge of 40 μC at each of its four corners. Determine the electric field at a point 5 m above the center of the square.

Solution: The distance $|R|$ between any of the charges and point P is

$$|R| = \sqrt{1^2 + 1^2 + 5^2} = \sqrt{27}.$$

$$\begin{aligned}
\mathbf{E} &= \frac{Q}{4\pi\varepsilon_0} \left[\frac{\mathbf{R}_1}{|\mathbf{R}|^3} + \frac{\mathbf{R}_2}{|\mathbf{R}|^3} + \frac{\mathbf{R}_3}{|\mathbf{R}|^3} + \frac{\mathbf{R}_4}{|\mathbf{R}|^3} \right] \\
&= \frac{Q}{4\pi\varepsilon_0} \left[\frac{-\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}}5}{(27)^{3/2}} + \frac{\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}}5}{(27)^{3/2}} + \frac{-\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}5}{(27)^{3/2}} + \frac{\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}5}{(27)^{3/2}} \right] \\
&= \hat{\mathbf{z}} \frac{5Q}{(27)^{3/2}\pi\varepsilon_0} = \hat{\mathbf{z}} \frac{5 \times 40\ \mu\text{C}}{(27)^{3/2}\pi\varepsilon_0} = \frac{1.42}{\pi\varepsilon_0} \times 10^{-6}\ (\text{V/m}) = \hat{\mathbf{z}} 51.2\ (\text{kV/m}).
\end{aligned}$$

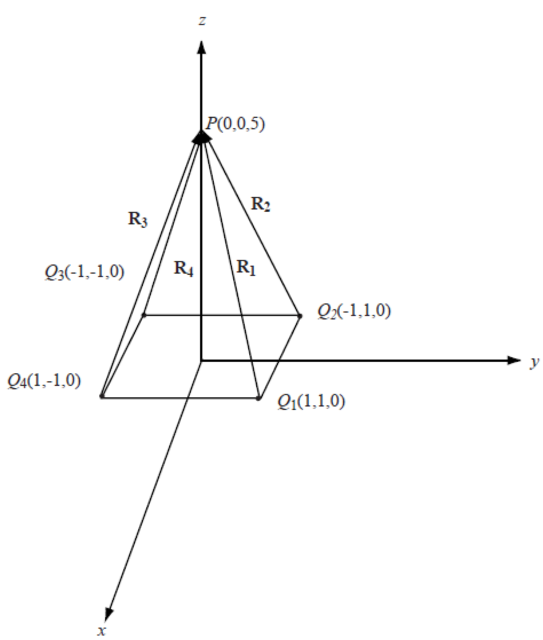


Figure P4.11: Square with charges at the corners.

Problem 4.17 Repeat Example 4-5 for the circular disk of charge of radius a, but in the present case, assume the surface charge density to vary with r as

$$\rho_s = \rho_{s0} r^2 \quad (\text{C/m}^2)$$

where ρ_{s0} is a constant.

Solution: We start with the expression for $d\mathbf{E}$ given in Example 4-5 but we replace ρ_s with $\rho_{s0} r^2$:

$$d\mathbf{E} = \hat{\mathbf{z}} \frac{h}{4\pi\varepsilon_0 (r^2+h^2)^{3/2}} (2\pi\rho_{s0} r^3\, dr),$$

$$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \int_0^a \frac{r^3\, dr}{(r^2+h^2)^{3/2}}.$$

To perform the integration, we use

$$R^2 = r^2 + h^2,$$

$$2R\, dR = 2r\, dr,$$

$$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \int_h^{(a^2+h^2)^{1/2}} \frac{(R^2-h^2)\, dR}{R^2}$$

$$= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \left[\int_h^{(a^2+h^2)^{1/2}} dR - \int_h^{(a^2+h^2)^{1/2}} \frac{h^2}{R^2}\, dR \right]$$

$$= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \left[\sqrt{a^2+h^2} + \frac{h^2}{\sqrt{a^2+h^2}} - 2h \right].$$

Problem 4.22 Given the electric flux density

$$\mathbf{D} = \hat{\mathbf{x}} 2(x+y) + \hat{\mathbf{y}}(3x-2y) \quad (\text{C/m}^2)$$

determine

(a) ρ_v by applying Eq. (4.26).

(b) The total charge Q enclosed in a cube 2 m on a side, located in the first octant with three of its sides coincident with the x-, y-, and z-axes and one of its corners at the origin.

(c) The total charge Q in the cube, obtained by applying Eq. (4.29).

Solution:

(a) By applying Eq. (4.26)

$$\rho_v = \nabla\cdot\mathbf{D} = \frac{\partial}{\partial x}(2x+2y) + \frac{\partial}{\partial y}(3x-2y) = 0.$$

(b) Integrate the charge density over the volume as in Eq. (4.27):

$$Q = \int_{\mathcal{V}} \nabla\cdot\mathbf{D}\, d\mathcal{V} = \int_{x=0}^{2}\int_{y=0}^{2}\int_{z=0}^{2} 0\, dx\, dy\, dz = 0.$$

(c) Apply Gauss' law to calculate the total charge from Eq. (4.29)

$$Q = \oint \mathbf{D}\cdot d\mathbf{s} = F_{\text{front}} + F_{\text{back}} + F_{\text{right}} + F_{\text{left}} + F_{\text{top}} + F_{\text{bottom}},$$

$$F_{\text{front}} = \int_{y=0}^{2}\int_{z=0}^{2} \left(\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)\right)\Big|_{x=2} \cdot (\hat{\mathbf{x}}\, dz\, dy)$$

$$= \int_{y=0}^{2}\int_{z=0}^{2} 2(x+y)\Big|_{x=2} dz\, dy = \left(2z\left(2y+\frac{1}{2}y^2\right)\Big|_{z=0}^{2} \right)\Bigg|_{y=0}^{2} = 24,$$

$$F_{\text{back}} = \int_{y=0}^{2}\int_{z=0}^{2} \left(\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)\right)\Big|_{x=0} \cdot (-\hat{\mathbf{x}}\, dz\, dy)$$

$$= -\int_{y=0}^{2}\int_{z=0}^{2} 2(x+y)\Big|_{x=0} dz\, dy = -\left(zy^2\Big|_{z=0}^{2} \right)\Bigg|_{y=0}^{2} = -8,$$

$$F_{\text{right}} = \int_{x=0}^{2}\int_{z=0}^{2} \left(\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)\right)\Big|_{y=2} \cdot (\hat{\mathbf{y}}\, dz\, dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2} (3x-2y)\Big|_{y=2} dz\, dx = \left(z\left(\frac{3}{2}x^2 - 4x\right)\Big|_{z=0}^{2} \right)\Bigg|_{x=0}^{2} = -4,$$

$$F_{\text{left}} = \int_{x=0}^{2}\int_{z=0}^{2} (\hat{\mathbf{x}}2(x+y)+\hat{\mathbf{y}}(3x-2y))\Big|_{y=0} \cdot (-\hat{\mathbf{y}}\,dz\,dx)$$

$$= -\int_{x=0}^{2}\int_{z=0}^{2} (3x-2y)\Big|_{y=0}\,dz\,dx = -\left(z\left(\frac{3}{2}x^2\right)\Big|_{z=0}^{2}\right)\Big|_{x=0}^{2} = -12,$$

$$F_{\text{top}} = \int_{x=0}^{2}\int_{z=0}^{2} (\hat{\mathbf{x}}2(x+y)+\hat{\mathbf{y}}(3x-2y))\Big|_{z=2} \cdot (\hat{\mathbf{z}}\,dy\,dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2} 0\Big|_{z=2}\,dy\,dx = 0,$$

$$F_{\text{bottom}} = \int_{x=0}^{2}\int_{z=0}^{2} (\hat{\mathbf{x}}2(x+y)+\hat{\mathbf{y}}(3x-2y))\Big|_{z=0} \cdot (\hat{\mathbf{z}}\,dy\,dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2} 0\Big|_{z=0}\,dy\,dx = 0.$$

Thus $Q = \oint \mathbf{D}\cdot d\mathbf{s} = 24-8-4-12+0+0 = 0.$

Problem 4.27 An infinitely long cylindrical shell extending between $r = 1$ m and $r = 3$ m contains a uniform charge density ρ_{v0}. Apply Gauss's law to find $\mathbf{D}$ in all regions.

Solution: For $r < 1$ m, $\mathbf{D} = 0$.

For $1 \le r \le 3$ m,

$$\oint_S \hat{\mathbf{r}} D_r \cdot d\mathbf{s} = Q,$$

$$D_r \cdot 2\pi r L = \rho_{v0}\cdot \pi L(r^2-1^2),$$

$$\mathbf{D} = \hat{\mathbf{r}} D_r = \hat{\mathbf{r}}\,\frac{\rho_{v0}\pi L(r^2-1)}{2\pi r L} = \hat{\mathbf{r}}\,\frac{\rho_{v0}(r^2-1)}{2r}, \qquad 1 \le r \le 3 \text{ m}.$$

For $r \ge 3$ m,

$$D_r \cdot 2\pi r L = \rho_{v0}\pi L(3^2-1^2) = 8\rho_{v0}\pi L,$$

$$\mathbf{D} = \hat{\mathbf{r}} D_r = \hat{\mathbf{r}}\,\frac{4\rho_{v0}}{r}, \qquad r \ge 3 \text{ m}.$$

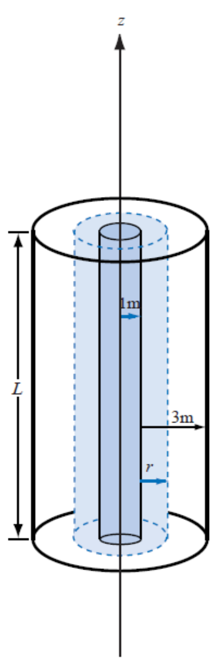


Figure P4.27: Cylindrical shell.

Problem 4.36 For each of the distributions of the electric potential V shown in Fig. P4.36, sketch the corresponding distribution of $\mathbf{E}$ (in all cases, the vertical axis is in volts and the horizontal axis is in meters).

Solution:

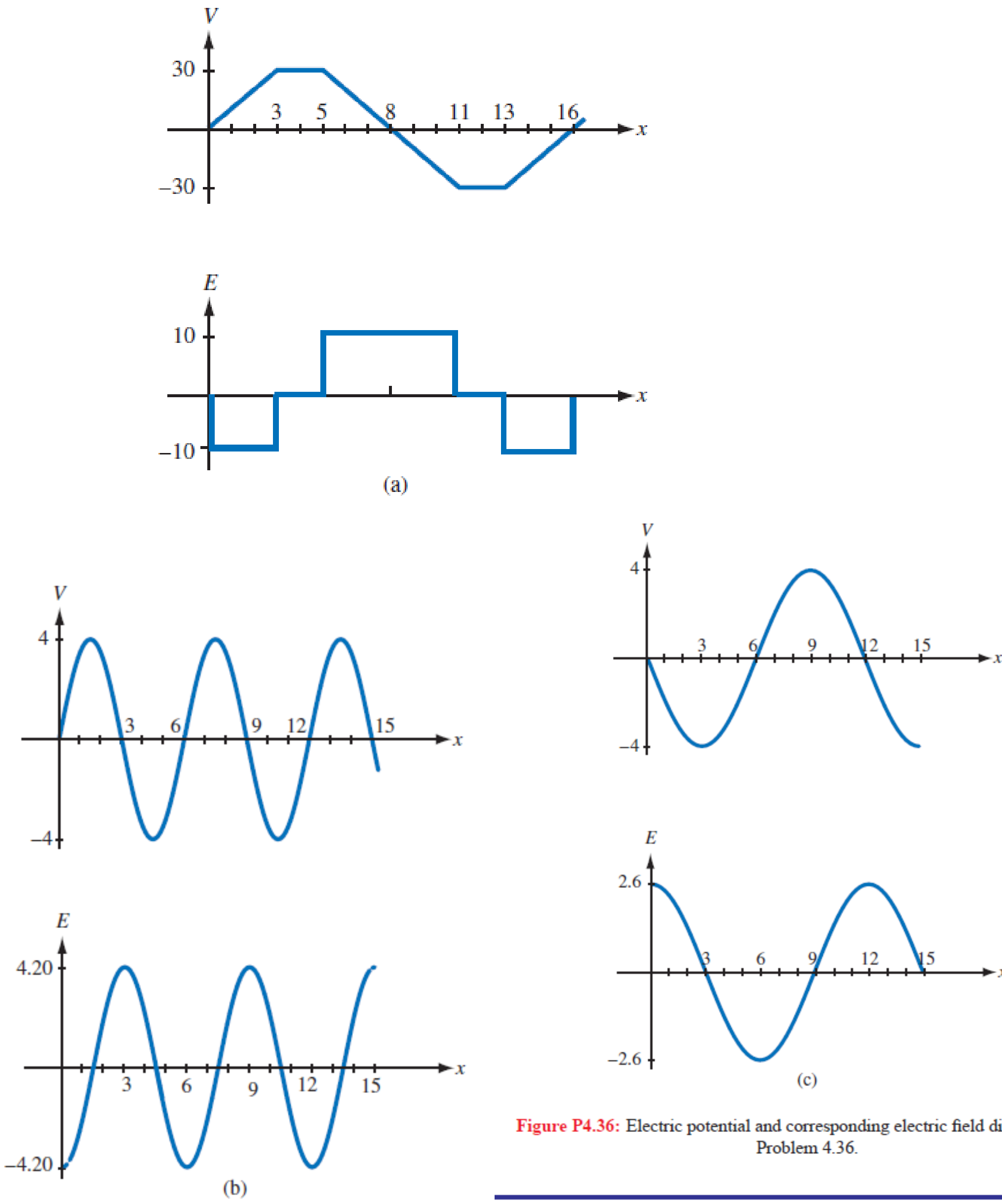


Figure P4.36: Electric potential and corresponding electric field distributions of Problem 4.36.